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An Absence of Elephants in the Room: Religion, Philosophy, and Negative Numbers in Albert Girard's A New Discovery in Algebra

Ethan Wilmes
Lawrence University

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Ethan Wilmes

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Professor Kern

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"Take away $\sqrt{3}$ from $\sqrt{6}$. There will remain $\sqrt{6}-\sqrt{3}$. Take away 2 from $\sqrt{5}$. There will remain $\sqrt{5}-2$. But if someone said, take away 7 from $\sqrt{48}$, this would be impossible; for $\sqrt{48}$ is less than 7. Nevertheless, the solution is $\sqrt{48}-7$."[2]

—Albert Girard, *A New Discovery in Algebra,* 1629*.*

Nevertheless, the solution is. With one seemingly indisputable claim, Albert Girard transformed a straightforward paragraph about the addition of roots of numbers into a challenge against widely held ideas about the existence of number. In early seventeenth-century Europe, the lines separating theology, science, and humanism were thin; what the modern reader understands as three distinct spheres of knowledge considerably overlapped with one another. Scientific discoveries and innovations coming from new technologies and foreign lands were laden with implications about theology and the human condition. While bland to all but the most fringe historians of mathematics today, the discovery of negative numbers led to a passionate and occasionally fierce epistemological debate throughout Europe. Falling outside of traditional mathematical knowledge, negative numbers found themselves in a sort of

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[1] Before discussing *A New Discovery in Algebra*, a note is needed. I do not speak French and as such, was dependent upon translations of this text in my analysis. I chose to use Black and Schmidt's 1986 translation of *Invention Nouvelle en l'Algèbre* found in their work *The Early Theory of Equations: On Their Nature and Constitution: Translations of Three Treatises*. Unfortunately for my analysis, their translation covers only the algebra portion of Girard's pamphlet. To compensate for this, I have worked through the non-algebra sections in the original French with the assistance of my friend Ezra Marker, who I am greatly indebted to. In combing these sections, we searched for anything that could be considered relevant to negative numbers, whether that be negatively signed quantities, or words like "moins," "absurde," "négatif," or "impossible." Upon finding these passages, Ezra attentively translated the individual sections where they appeared, as well as the entirety of Girard's dedication. To differentiate between the two translations in the footnotes, "A New Discovery in Algebra" will refer to the Black and Schmidt translation, while "*Invention Nouvelle en l'Algèbre*" will refer to translations done by Ezra Marker.

[2] Albert Girard, "A New Discovery in Algebra," in *The Early Theory of Equations: On Their Nature and Constitution: Translations of Three Treatises*, trans. Ellen Black and Robert Schmidt, (Annapolis, MD: Golden Hind Press, 1986), 114.

existential limbo; however useful they proved themselves to be, their reality was regularly denied by the united forces of antiquity, Catholicism, and contemporary mathematics. In their efforts to share their new discoveries while constrained by these pressures, Early Modern mathematicians like Albert Girard were implicitly required to serve as mediators in order to formalize and publish their discoveries. While his pamphlet *A New Discovery in Algebra* seems to be full of contradictions, evasions, and ambiguities, these apparent oversights were in fact tools that allowed Girard to maneuver around obstacles which hampered the spread of discoveries among Europe's developing community of mathematicians.

You're a Lutenist and a Scholar, Albert.

Our story begins in 1595, Saint Mihiel, France. There, Albert Girard was born to Protestant parents, likely Calvinists. Unfortunately for historians of mathematics, Girard left a scant biographical record. He did not keep a journal, and most of our knowledge of him comes from forwards, dedications, and letters to prominent early scientists who used a blend of religion and metaphysics to understand the world, known as natural philosophers.[3] Early in his life he and his family resettled in Holland, presumedly as refugees from the Wars of Religion taking place in France (1562–1598). We know that he studied at the University of Leiden, and later made ends meet serving as a military engineer in the army of Frederick Henry, Prince of Orange, and professionally playing the lute.[4] Following his death in Leiden in 1632, Albert Girard's widow wrote that his passing left her and their 11 children destitute, with "only his reputation of having faithfully served and having spent all his time on research on the most noble secrets of mathematics."[5] This cannot be understated: although Girard died a poor man, his contributions to mathematics are still felt today.

[3] "Albert Girard – Biography," Maths History, accessed March 10, 2022, https://mathshistory.st-andrews.ac.uk/Biographies/Girard_Albert/.
[4] Ibid.
[5] Jean Itard, "Girard, Albert," Encyclopedia.com, April 26, 2022, https://www.encyclopedia.com/science/dictionaries-thesauruses-pictures-and-press-releases/girard-albert.

Alongside his work on the fundamental theorem of algebra that will be discussed later, Girard influenced many different areas of mathematics. In his work *Trigonométrie* (1626), he was the first to abbreviate sine, cosine, and tangent to sin, cos, and tan that we use today.[6] The modern notation for the cube root, $\sqrt[3]{}$, was Girard's as well, and he was also the first recorded mathematician to come up with the inductive definition of the Fibonacci numbers. Regrettably for his status as a mathematician, Albert Girard's name doesn't often appear in the list of the great innovators. Generally stingy with his ink, he was often disinclined to write out the proofs or diagrams that are valued in the Greco–Euro tradition. Alongside this, he wasn't what many would consider a *modern* mathematician. Prominently missing in *A New Discovery in Algebra* is notation for the quantity 0, as well as general coefficients for variables. While many of his notations were picked up by later mathematicians, not all of them stuck, and the syncopated algebra he used was abandoned for the wholly symbolic algebra that we know today.[7] Regardless of the critiques that mathematicians may levy on him, Albert Girard is a fascinating and essential character to the development of today's mathematics.

The Book of Math: Religion and Reception

While contemporary practitioners prefer to conduct their study of mathematics in a stable and secular space, the mathematics of Albert Girard developed in a turbulent and religious world. As a Huguenot refugee of the Wars of Religion, our protagonist was no stranger to religious persecution. His time served as a military engineer in the Anglo–Spanish War (1625-1630) would have further emphasized both the physical and religious importance of his beliefs. Heated public debates about the legitimacy of

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[6] University of St. Andrews, "Albert Girard – Biography."

[7] The history of algebraic notation is regularly split into three periods, those of rhetorical, syncopated, and symbolic algebra. To demonstrate the differences of these algebras, we'll use $x^2 + 2x = 4$ to show differences in notation. Rhetorical algebra is the practice of writing equations using only words. In rhetorical algebra, our example equation could potentially be written as "when one quantity squared plus double the original quantity is equal to four." Syncopated algebra uses a mix of words and symbols, so one expression of our example could be "when 1 x squared plus 2 x is equal to 4." Symbolic algebra is used in today's mathematics, so our example equation would be presented as it was first presented.

Protestantism served as a constant reminder to all that eternal damnation could be right around the corner. Girard himself personally engaged in this debate, accusing the Catholic soldier Honorat de Meynier of attacking "those of the Reformed religion by calling them heretics."[8]

But why should we pay attention to religion in a pamphlet about algebra? As will be further discussed, besides being a useful discipline for solving real world problems, mathematics was deeply connected with theology and religion. Additionally, everyone in Europe at the time of *A New Discovery in Algebra's* publication was a dedicated member of their respective religion. While the Protestant Reformation had shaken the authority and power that the Catholic Church held over Europeans, it was still an active and imposing figure in both the political and religious worlds. Even though Albert Girard was a Protestant, he would have to couch his claims within the Catholic Church's tradition of speculation to assure his work would be accessible to his readers across Europe. While commonly misunderstood as an expression of some eternal struggle between tradition and innovation, speculation is much more nuanced. New scientific theories and discoveries were not inherently contrary to doctrine or anathema; it was possible to propose theories that directly contested scripture. The important part for the Catholic Church was not *what* was argued, but instead the apparent *conviction* of what was argued. Claims could be made denying what the Bible said about the motion of the planets, the age of the Earth, or the building blocks of existence, so long as these claims were couched in speculation.[9] But even as Girard's Calvinism freed him to personally embrace ideas that contradicted the Catholic Church's dogma, the public nature of his work bound him to operate by the Pope's rules, since works could be declared anathema or barred from publication. Like any scientist sharing their work, transmission was key: Girard wanted to make sure his pamphlet could physically reach other scientists, and once there, it was essential that his message would not fall on deaf ears.

[8] Itard, "Girard, Albert."
[9] Lisa Jardine, *Ingenious Pursuits: Building the Scientific Revolution,* (New York: Anchor Books, 1999), 9.

We must take care to neither over- nor under-dramatize the stakes of his pamphlet. By the 1630s, Confessional Europe was increasingly coming to terms with the idea of a Europe made of multiple Christianities. While it's clear that the position of the Protestants was still controversial in the eyes of Catholics and that mob violence would spring up time to time, Girard was not at risk of any bodily harm for his pamphlet on algebra for two reasons. The first is that negative numbers simply fell outside of traditional Christian discourse. It is a process of historical fiction to imagine if and when negative numbers could have come into contact with the fathers of the Church. The main issue here is conceptualization. The Bible discusses various ideas about debt, which is commonly used as an introductory metaphor for negative numbers. From a cross-cultural vantage point, we can find earlier examples of Chinese and Indian mathematicians understanding of negative numbers as "losses" and "debts," respectively.[10] To consider the discussion and use of debts as negative numbers, however, is tenuous and likely anachronistic. Debts can be understood as differences: If Alex owes Barbra 10 cows, and Barbra owes Alex 4 cows, Alex and Barbra don't need to use negative numbers—they can simply look at who owes the other more cows, and then find the difference between the two quantities, avoiding any conception of negative numbers, or even 0. Alex simply owes Barbra the (positive) quantity of 6 cows. Within this exercise of historical imagination, even if the Church fathers did hypothetically understand negative numbers as numbers, they also would have needed to consider whether they were germane to the Bible; as it will be discussed later, there are no explicit references to negative number in the Bible, further solidifying negative numbers as a specific historical idea which postdates the writing of the Bible. The other protection afforded to Girard came in the form of the regional and chronological placement of his publication. While ideally, he would have wanted his pamphlet to reach audiences throughout Europe, Girard himself was still physically within Holland. Church officials at the time were more concerned with managing the spread of controversial

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[10] David Mumford, "What's so Baffling about Negative Numbers? — A Cross-Cultural Comparison," in *Studies in the History of Indian Mathematics*, ed. C. S. Seshadri (Gurgaon, India: Hindustan Book Agency, 2010), 121–123.

works than they were with punishing the authors of these works. Alongside this, there is a question of infrastructure: Even if some overly zealous Catholic from Spain, Southern France, or the Vatican got ahold of Girard's work and deemed it sufficiently controversial to suppress, Girard was still within a foreign land where the Church held little power. This complicates a discussion of the stakes. Girard's were in the spread of his fundamental theorem of algebra. The "opposing" Catholic Church held no direct qualms with his discovery, but instead was focused on legitimization and control of discourse. Between these two actors were Girard's readers, who occupied the middle ground of wanting to read Girard's work while also needing to adhere to their specific religious doctrines. Thus, Girard's story is not one of a rebel scientist facing off against an antiquated religion, but instead a conversation happening within a single text, as these potentially opposing parties attempt to come to an agreement on how this work can be disseminated.

A Frenchman in Holland

Although Girard left France very early on and spent a vast majority of his life in the Netherlands, he wrote for a French audience. While his dedication states that his pamphlet was written for a calvary captain of the Netherlands, greater material, historical, and biographical information lead us to believe that Girard also intended for the work to reach other readers. For starters, *A New Discovery in Algebra* is a printed work. If Girard wanted to only share his ideas with the calvary captain, he wouldn't have gone through the work and investment required to publish his thoughts. Additionally, France was the center of mathematics in the seventeenth-century, evident by the "household" names of Pascal, Fermat, and Descartes. Many of his works were translations from Flemish into French, including that of his friend and fellow military engineer Simon Stevin's *Arithmetic* (1625), as well as Hendrik Hondius' *Treatise on Fortifications* (1625).[11] His own works were also written in French and were referenced by various

[11] University of St. Andrews, "Albert Girard – Biography."

Frenchmen. Moreover, Girard never considered the Netherlands his own country, and felt grief about not living in his homeland due to the persecution of Protestants.[12] His yearning for his homeland of France, his writing in French on a subject centered in France, and his frequent translations of works for the French make it clear that while he lived in the Netherlands, the work was markedly by a Frenchman and largely for Frenchmen.

What is Math?

Before diving into a discussion of numbers, negative or otherwise, we first ought to define them—not mathematically, but metaphysically. One view of the metaphysics of numbers, mathematical realism "is the philosophical position that mathematical statements such as 'there are infinitely many prime numbers' are true and that these statements are true by virtue of the existence of mathematical objects–prime numbers, in this case."[13] A prominent alternative to realism is mathematical fictionalism, that is ". . . is the view that mathematical statements, such as '$7 + 5 = 12$' and 'π is irrational' are to be interpreted at face value and, thus interpreted, are false . . . because these statements imply the existence of mathematical entities, and, according to fictionalists, there are no such entities."[14] This is not to say that math is completely meaningless, but rather, to derive truth from math, one must accept a fictional "story of mathematics," a story of objects that do not exist outside of the human imagination.[15] While the discussion of the source of mathematical truth is quite interesting, more important to this paper is that of the existence of mathematical objects. For my analysis, I choose to adopt the lens of fictionalism in order to emphasize the transmission of Girard's unorthodox ideas. The treatment of mathematical objects as *ideas* instead of extant *objects* allows us to properly historicize negative numbers in their specific cultural

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[12] Itard, "Girard, Albert."
[13] Mark Colyvan. *An Introduction to the Philosophy of Mathematics*. (New York, NY: Cambridge University Press, 2012) 36.
[14] Ibid., 55.
[15] Ibid., 56.

context. The assumptions of mathematical realism take the existence of number for granted. If negative numbers have always existed, it seems that their "discovery" and subsequent spread would have been inevitable, only being opposed by backwards looking traditionalists. This philosophical view of the number tends to lead to a triumphalist view of the Scientific Revolution commonly reproduced within internalist histories of science, one that eliminates many of the specific contingencies that led to the acceptance of negative numbers that we see today. By considering numbers and mathematical relationships as ideas operating within a specific historical discourse instead of fixed objects, our study of *A New Discovery of Algebra* departs from these reductionist internalist histories. Through fictionalism, we are able to better focus on how negative numbers spread, their discursive forms, how they evolved, and most importantly, how a "universal truth" came to be.

While a fictionalist understanding of the number is important for the historical analysis of *A New Discovery in Algebra*, we must also consider how Girard himself understood number.[16] Although he does not explicitly state his philosophical views, Girard implicitly positions himself as mathematical realist, and provides a physicalist interpretation of negative numbers. Physicalism is perhaps best understood in juxtaposition with Platonism, another form of mathematical realism. Perhaps the most hardline stance on realism, Platonism locates the abstract objects of mathematics on an inaccessible plane, where every possible mathematical object that could exist does.[17] More fixed in the material world that surround us is physicalism. This strand of mathematical realism argues that mathematical entities are not intangible, but instead can be found all around us in the physical world. To borrow from philosopher Penelope Maddy physicalism can be experienced in day-to-day life, as ". . . every time you look in the refrigerator and see a dozen eggs you are seeing the set of 12 eggs. You are thus face to face with a mathematical object,

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[16] It is difficult to distill an individual mathematician's beliefs into a singular taxonomic category of philosophy, and impossible to do so for an entire amalgam of mathematicians located throughout Europe; nevertheless, what is lost in reduction will be offset by what is gained in our conceptual framework.
[17] Ibid., 38.

namely a set."[18] During Girard's time, European mathematicians embraced various philosophies of mathematics, from the Platonic views inherited from the Ancient Greeks to the rationalist and empirical thoughts emerging during the beginning of the Enlightenment (ca. 1637–ca. 1804).[19] In the primary translation of the text that this paper uses, Black and Schmidt have taken the liberty to emphasize the realist position, translating the French "invention" to the English "discovery," instead of the more obvious English cognate "invention." While "invention" carries the message that Girard's ideas were *created* by himself, "discovery" highlights the beliefs that he and his audience would hold about his work, that he had *found* something that had been there all along.

A helpful section for understanding Girard's philosophy of numbers is his discussion of incommensurables and writing math problems.[20] In providing examples of how these numbers should be added, Girard clams that "Heterogenous things, or those of diverse nature must not combine. Thus, wood and iron do not combine. In geometry lines do not enter in comparison with surfaces. And incommensurable numbers cannot combine in number by addition or by subtraction (although they can in geometry)."[21] We can see here how Girard emphasizes not the existence of these posited mathematical objects, but instead how an underlying categorical structure fixed in the physical world shapes how they can interact. He further reveals his realist ideology, stating "… 6 oxen, 8 sheep, and 2 camels are 16 animals; for this case it is necessary to give to the sum the name of the nearest genus that comprehends

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[18] Ibid., 37.
[19] David Nirenberg and Ricardo L. Nirenberg, *Uncountable: A Philosophical History of Number and Humanity from Antiquity to the Present*. (Chicago, IL: University of Chicago Press, 2021.), 105–110, 134–146.
[20] Numbers are said to be commensurable if their ratio $\frac{a}{b}$ is a rational number, otherwise they are incommensurable. Without going into too much detail and making this a math paper, a footnote of explanation is provided. A number is said to be rational if it can be expressed as a fraction using only whole numbers. Some examples include 2.5 ($\frac{5}{2}$), $.\overline{7}$, ($\frac{7}{9}$) and 63 ($\frac{63}{1}$). Irrationals on the other hand, are numbers that cannot be expressed as fractions, like $\sqrt{2}$ or π. Some examples of pairs of commensurable numbers are 4 and 5, since their ratio $\frac{4}{5}$ is rational, as 7π and 2π, as $\frac{7\pi}{2\pi} = \frac{7}{2}$, which is rational, or $9\sqrt{3}$ and $\frac{1}{2}\sqrt{3}$, whose ratio is $\frac{18}{1}$. Some examples of incommensurable pairs are π and 12, $\sqrt{23}$ and 9, or e and $\sqrt{7}$.
[21] Albert Girard, *A New Discovery in Algebra*, (Annapolis, MD: Golden Hind Press, 1986), 113.

such species."[22] The idea of the nearest genus highlights that there is some underlying, preexisting concept that relates these objects, linking and allowing them to be connected by number. Though Girard acknowledges that these numbers can be found between objects in the physical world, he also notes the value of abstraction in mathematics when discussing false positing, a method for solving equations. As part of this method ". . . to resolve a question, it is necessary to once again set in question some abstract number, without (if one can manage it) speaking of a material like crowns, feet, etc."[23] While Girard believes that it is possible to find examples of numeric structure in the physical world, he prefers to preserve the abstractness of underlying structure that makes mathematics so useful. Most important for this paper, is that his physicalism towards negative and positive numbers. At the very end of the algebra section, he states that "The minus solution is explicated in Geometry by retrograding; the minus goes backwards where the + advances."[24] He is able to find positive and negative numbers not on the Platonic plane, but by choosing a direction and moving backwards. Although both Platonism and physicalism fall under the broad umbrella of mathematical realism, Girard's physicalist view granted him the ability to engage with numbers and the world around him than his negative denying peers.

A Note on Notation

Before turning an eye to the circumstances which shaped *A New Discovery in Algebra*, some notes are required on the presentation of mathematics in this paper. Mathematical notation has changed in many ways since Girard's time, so the exhibition of formulas and equations in his pamphlet varies greatly from how the same formulas and equations would be presented now. It has been discussed above that Girard used the syncopated algebra of his time, which varies from the symbolic algebra we use today. His ordering conventions of equations are also different, as Girard normally places the variable of the greatest

[22] Ibid., 113.
[23] Ibid., 122.
[24] Ibid., 145.

power alone on the left, with the remaining terms being placed on the right. Additionally, the symbols he uses to represent variables and exponents are nearly unrecognizable without being introduced to the reader beforehand. These conventions are best explained by demonstration. In order to discuss what we would write as $x^5 - 4x^2 + 2 = 0$, Girard would write "when $1⑤$ is equal to $4② - 2$."[25] He also doesn't use modern terms for certain mathematical concepts, like quartic ("square square"), zero ("nothing"), or negative ("less than nothing"). For these and similar cases from Girard and other earlier mathematicians, I have taken the liberty of "translating" their ideas and notations into formats more familiar for my readers. I acknowledge that these different notations are not simply different names of the same idea but do have their own meanings and connotations relative to the circumstances in which they were written. Besides notation issues, I recognize that all of my readers may not be comfortable with some of the math Girard employs. To account for this, I have included brief explanations of the relevant mathematical concepts in footnotes as they appear. For the sake of brevity and scope of this paper, I will only be focusing on Girard's thoughts on negative and complex numbers, although his pamphlet could be read with a different lens to further shed light on how he and his readers understood their world.

The Divinity of Numbers: Christian Influences on Numeric Discourse

Number in the fifteenth- through seventeenth-centuries was not simply associated with quality, but also carried symbolic significance. In his essay "Mystical Arithmetic in the Renaissance: from Biblical Hermeneutics to a Philosophical Tool," Jean-Pierre Brach defines mystical arithmetic as "a type of understanding of mathematics that imbues numbers with the capacity of signifying more than just the quantity they [the numbers] materially refer to."[26] Two modern, secular examples of this include the association of 7 and 13 with luck and misfortune, respectively. In the tradition of Catholic hermeneutics,

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[25] Here, the circled numbers denote the power of the variables. 2(8) would denote $2x^8$.
[26] Jean-Pierre Brach, "Mystical Arithmetic in the Renaissance: from Biblical Hermeneutics to a Philosophical Tool," In *Mathematicians and Their Gods: Interactions between Mathematics and Religious Beliefs*, ed. Snezana Lawrence and Mark McCartney (Oxford: Oxford University Press, 2015), 105.

numerology was applied to the Bible, granting any number that appeared within its religious significance, through the specific practice of arithmology. Interestingly, it seems there was a sort of isomorphism within arithmology between numbers and their religious referents. Brach provides the example that since 3 is associated with the Holy Trinity and 4 is associated with the four corners of the world, 12 (3 multiplied by 4) is associated with bringing Gospel to the 4 corners of the world.[27] Catholic arithmology gained many observants throughout Europe, evident in the numerous arithmology books printed and circulated throughout the Early Modern Period following the creation of the printing press.[28] Importantly, Catholic arithmology only focuses on the positive integers, that is to say 1, 2, 3, 4,… onwards, with particular reverence placed on 1 and the primes.[29] While the exclusion of irrationals (π, $\sqrt{3}$), negatives, complex numbers ($\sqrt{-4}, 12i$), and fractions seems obvious, (outside of ½ for King Solomon, of course) the important result from Catholic arithmology is an implied hierarchy of numbers, which placed the positive integers over other types of numbers in both exposure and sanctity. Though arithmological discourse was widespread throughout Europe, not every European Christian was able to embrace it. Where Catholics used the Bible to solve a multitude of questions, many that modern readers wouldn't consider religious at all, Protestants treated the Bible as the answer to a single question: "How can sinful human beings be saved from the everlasting fires of hell?"[30] These differing goals of the Bible were in line with the greater religious beliefs on the groups following its divine word; Catholics could interpret the Bible metaphorically to solve a variety of questions, while Protestants took its words literally in an attempt to achieve eternal salvation through God's grace.

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[27] Ibid., 107.
[28] Ibid., 109–116.
[29] John J. Davis, *Biblical Numerology: A Basic Study of the Use of Numbers in the Bible* (Grand Rapids, MI: Baker Book House Company, 1992.)
[30] James D. Tracy, *Europe's Reformations 1450–1650* (Lanham, MD: Rowman and Littlefield Publishers, Inc., 1999), 13.

Even more important to religious-mathematical discourse was the tradition of Catholic theologian-mathematicians. Dating back to the first century, Platonic ideas on the existence of the number persevered and were used within theological debates on Abrahamic religion in Philo of Alexandria's (ca. 20 BCE–ca. 50 CE) *On the Creation* and *Questions and Answers on Genesis.*[31] Early in the Christian discourse, well known figures such as Ptolemy (ca. 100–ca. 170 CE) and Valentinus (ca. 100–ca. 180 CE) were accused in catalogs of error of being "'false' apostles of arithmetic."[32] This claim was brought to them on grounds that were similar to the arithmological isomorphism discussed above. When counting the natural numbers, those being 1, 2, 3, … onwards until we choose to stop, we find ourselves adding 1 to itself repeatedly. The generation of many numbers from a single entity was one of the important metaphysical questions that these writers tried to understand: How could a singular God create mortal multitudes, and how could humans recreate unity to achieve salvation?[33] Looking away from these "'false' apostles," we also find Saint Augustine of Hippo (354–430 CE) as an example of a figure within the Church engaging with mathematics. Tying religion and mathematical thought together, Augustine argued that we can find evidence of God in our ability to understand numbers.[34] In his eyes, number was a divine Platonic concept that humans would never be able to access on their own—yet they did. He reasoned that our ability to understand numbers must have been imparted to humans by God, proving his existence. Dating back to early Christian discourse, we can find a plethora of examples showing that numbers were not just characters imbued with quantitative and symbolic meaning, but an important part of discourse on God and the universe.

Negated Negative Numbers and Convoluted Complex Constants

[31] Nirenberg and Nirenberg, Uncountable, 102–104.
[32] Ibid., 106–107.
[33] Ibid.
[34] Ibid., 108.

Alongside Christianity, a veneration of texts from Greek antiquity and contemporary work being done as part of the Scientific Revolution (ca. 1550–ca. 1700 CE) influenced discourse on negative and complex numbers in Early Modern Europe (ca. 1500–ca. 1800 CE). While these strands of knowledge were intimately intertwined, it's important to parse them out and identify which specific beliefs helped shape discourse. Once these individual contributions are recognized and contextualized, it becomes much easier to see the world through the eyes of a seventeenth-century European mathematician and rationalize their reasons for not engaging with these numbers.

Essential to understanding Catholic natural philosophers during the Scientific Revolution is their belief in the two books of God. This belief stated that God provided man with two "books" to learn from, one of scripture and the other of nature. The book of nature consisted of the wonders of the physical world, from the microscopic organisms seen swimming in pepper water to the heavenly motions of the planets around the sun. Using arguments based on these perceived miracles to prove the existence of God, natural philosophers applied the language of mathematical and scientific discourse to religion. This discourse was further enforced by the institutional support of medieval universities, who required theologians to be proficient in mathematics, logic, and natural philosophy.[35] Because of this book of nature, any scientific discovery was packed with theological results, varying from celebrations of God's power to issues with biblical chronologies of the earth. Much like in the acceptance of arithmology, Protestants differed from Catholics. Although plenty of natural philosophers were Protestants, their belief in *sola scriptura*, that divine revelation comes only from the Bible, led to a separation of theology and number. While Catholics studied the natural world to better understand God, Protestants' understanding of the natural world was accordingly much more secular. This importance of divinity in the natural world led to some issues with negative and complex numbers among Catholics. There's no such thing as a

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[35] Lawrence M. Principe, *The Scientific Revolution: A Very Short Introduction* (Oxford, UK: Oxford University Press, 2011), 8.

negative cell to examine under a microscope, nor a negative rock that you can drop and record its position, speed, and acceleration—and that is to say nothing of complex numbers. The lack of a physical, negative reality leads to the ecclesiastical question that is central to this paper: If Catholics are unable to find negative numbers in either scripture or the natural world, can they truly be a part of the “divine science?”

Divinity was not the only thing acting on European’s ideas, or the lack thereof, on negative numbers. Fueling Renaissance (ca. 1300–ca. 1600 CE) ideas on the rebirth of Europe was the rediscovery, translation, and circulation of many Ancient Greek (ca. 800 BCE–600 CE) texts. These texts and their ideas were widely celebrated among Europeans and were responsible for three important factors influencing the rejection of negative numbers in Early Modern Europe: a mathematical fascination with geometry, a lack of conception of negative numbers within the mathematical canon, and a rejection of non–Ancient Greek texts.

Let us first examine the fascination on geometry as a method for knowing truth. Although Galileo Galilei (1564–1642) participated primarily in the Italian mathematic sphere, his outlook presented *The Assayer* is emblematic for the time. He claims:

> Philosophy is written in this grand book — I mean the universe — which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the language in which it was written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a word of it; without these one is wandering around in a dark labyrinth.[36]

[36] Mark McCartney, “Introduction,” In *Mathematicians and Their Gods: Interactions between Mathematics and Religious Beliefs*, ed. Snezana Lawrence and Mark McCartney (Oxford: Oxford University Press, 2015), 2.

Alongside a reference to the book of nature, Galileo makes the claim that the knowledge of geometry is essential to understanding the world. Until the late 1600s, translations of ancient mathematical texts focusing on geometry, particularly Euclid's *Elements* (ca. 300 BCE), held a privileged position within the European mathematical canon. The first notable attack on *Elements* in French was initiated by Petrus Ramus, (1515–1572) who was later followed by Antoine Arnauld (1612–1694) and Jean Prestet (1648–1690).[37] These three authors released textbooks criticizing the structure and content of the books of *Elements*, leading to its eventual fall from grace. The geometric thinking found in *Elements* tended to be inconducive, if not wholly incompatible with the development of negative numbers. This incongruency stemmed from the Ancient Greco–conception of the number which was based on the ratio of lengths of lines, instead of an additive or self-evident existence. For instance, Antoine Arnauld took issue with the ratio $1{:}-1$, noting that it is equivalent to $-1{:}1$.[38] If these two ratios were equal, Arnauld reasoned, then -1 was simultaneously less than and greater than 1.[39] His objection is one of many that highlights how the inheritance of geometrical thought created a barrier that stifled the conception of an abstract theory of negative numbers in European thought.

The second issue stems from the geometric issues discussed above, that the Ancient Greeks provided later mathematicians with no examples of negative numbers to work from. Nearly every history of mathematics textbook will include the excerpt from Diophantus' *Arithmetica* (ca. 250 AD), in which he

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[37] Gert Schubring, *Conflicts between Generalization, Rigor, and Intuition: Number Concepts Underlying the Development of Analysis in 17th–19th Century France and Germany*, (New York, NY: Springer New York, 2010), 67–70.

[38] For those less comfortable with mathematics, ratios are essentially comparisons between two numbers based on their value. When dealing with ratios, their relationship is preserved through multiplication, so $2{:}3$ is the same ratio as $4{:}6$. When multiplying ratios where both sides are positive by positive numbers, the number that is greater will always be on the same side. If we take the ratio $2{:}3$, and multiply it by some positive number x, we'll have $2x{:}3x$, and the $3x$ will always be greater than the $2x$. But this focus on greater and less becomes convoluted if negatives are involved. In the Arnauld example, we have $1{:}-1$, where the right side is less than the left side. When we multiply it by -1, the relationship between the two numbers is preserved, but we now have $-1{:}1$, where the left side is less than the right side.

[39] Schubring, *Conflicts between Generalization, Rigor, and Intuition*, 55–57.

attempts to solve the equation $4 = 4x + 20$. While the modern reader (hopefully) sees that $x = -4$, Diophantus is unable to solve this equation, and dramatically calls the answer "absurd."[40] With his dismissal of negative numbers, later interpreters of the text who took Diophantus at his word were primed to carry his conceptions on the lack of negative numbers onwards. Notably, *Arithmetica* finds itself as the beginning of a long line of discourse that deems certain numbers illogical, which continues even today with the terminology of "imaginary" or "complex" numbers.

The rejection of mathematics from contemporary sources is yet another result of the Renaissance fixation on Ancient Greece. The exaltation of ancient Greek works came with an equal dismissal of many works that did not originate from them. Most notably, this included an attack on Arabic texts, with humanists criticizing Arabic influences found in translations of ancient works.[41] Geography and culture were not the only justifications for rejecting texts and ideas—earlier European texts were selectively picked through with a bias not applied towards those of the Classical Greeks. We can find an inclination against works of the Middle Ages (ca. 500–ca. 1300 CE) by looking at Leonardo de Pisa, (ca. 1170–ca. 1250 CE) commonly known as Fibonacci, who accepted some negative solutions in his manuscript *Liber Abaci* (1202 CE).[42] While the work's main claim to fame is the popularization of Arabic numerals in non-Islamic Europe, *Liber Abaci* was equally revolutionary because it is the first known European work to accept negative number solutions.[43] If Fibonacci deemed it appropriate, negative number solutions were

[40] Ibid., 37.
[41] Principe, *The Scientific Revolution: A Very Short Introduction*, 10–11.
[42] An interesting topic that I've yet to see explored could involve the juxtaposition of negative numbers and Arabic numerals. Why did Fibonacci's notation catch on while his negative numbers did not? Could it be because of the narrow situations when he did accept negative numbers?
[43] Schubring, *Conflicts between Generalization, Rigor, and Intuition*, 39. While Schubring makes the claim that *Liber Abaci* brought Arabic numerals to Europe, this is somewhat problematic. In her book *The History of Mathematics: A Very Short* Introduction, Jacqueline Stedall notes that Arabic numerals had been used in Europe by Muslims in the Iberian Peninsula for two centuries before Fibonacci published *Liber Abaci*.

accepted when the physical constraints of the problem would allow them.[44] Though there's a plethora of historical questions that can be posed using this work, most pressing is the counterfactual "why were negative numbers not widely adopted in the European tradition of mathematics, when examples of them had been present for over 350 years?"

Although I have emphasized the importance of Antiquity in the buildup to seventeenth-century mathematics, one last note is needed. As the Early Modern Period progressed, Europeans pridefully identified more and more with their contemporaries than the ancients they idolized during the Renaissance. This paper engages a work published in 1629, where both Renaissance and Early Modern ideas influenced culture. Because of these greater historical trends, the significance of Greco– mathematics in the European tradition came under growing criticism. Mentioned above, Arnauld, Prestet, and Ramus are just three of the many authors who attempted to make space for themselves in mathematics, at the expense of figures like Euclid and Archimedes. This is not to claim that there was some great triumph of "true" mathematical knowledge, but instead to emphasize the currents that were beginning to weaken Renaissance, and therefore Ancient Greek culture, in mathematics.

Girard and his contemporary mathematicians therefore found themselves somewhat free to interpret negative and complex numbers as they wanted. Where Euclid and the Catholic arithmologists did not reference the existence of any of negative numbers, other Ancient authors such as Diophantus clearly expressed inherent objections to them. Even though these traditional sources did not explicitly define what negative numbers were, they set some faint implicit barriers on what they might not be. The lack of a unified idea of what negative and complex numbers were led to a variety of conjectures regarding what these numbers might be. As discussed above, Fibonacci accepted negative numbers based on the

[44] Ibid. If the problem engaged with monetary exchange, for example, he would accept negative solutions, but if the problem focused on something that could not be interpreted with negatives, the problem would be considered unsolvable.

qualitative properties of the math problem.[45] Michael Stifel (1487–1567) followed in the steps of Diophantus and termed them "numeri absurdi."[46] Gerolamo Cardano (1501–1576) disputed the law of signs, claiming a negative multiplied by a negative resulted in a negative product.[47] François Viète (1540–1603) didn't accept negative solutions to equations at all.[48] Simon Stevin (1548–1620) accepted negative solutions but prioritized positive ones.[49] Our protagonist, Albert Girard found himself amidst a whirlwind of debate on the existence, value, and significance of negative numbers, where almost anything could go.

But what did the intersection of varying Christian interpretations of number, Greek antiquity and contemporary scholarship look like in practice?

The Monotheistic Convergence Theorem

To get an in depth understanding of how Catholics interacted with negative numbers, we look to Girard's contemporary René Descartes. While he may not be the flagbearer of Catholicism, evident by his proof of God that the Church was less than enthusiastic about, he is an excellent litmus test to evaluate Catholic views of negative numbers. If the Church–antagonizing Descartes wasn't willing to embrace negative numbers, then it's doubtful that the Church's and its more reserved and orthodox Catholic mathematicians would have accepted them at face value.

One of Descartes's issues with negative numbers can be viewed in the language he uses to talk about negatives. When referring to negative numbers, he applies what I define as *anti-existence terminology*.[50] Stemming from discourse of mathematical realism that treats numbers as existing objects, this terminology describes certain numbers as fake, fictive, false, absurd, or any other terms which deny

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[45] Schubring, *Conflicts between Generalization, Rigor, and Intuition*, 39.
[46] Ibid., 41.
[47] Ibid., 42–45.
[48] Ibid., 46.
[49] Ibid.
[50] Ibid., 47.

their "reality." In Descartes' *La Géométrie* (1637CE), he refers to negative solutions as "false roots," while positive solutions are "true roots."[51] Notably, he goes a step further than standard anti-existence terminology, using the phrase "defect of a quantity" to describe negative numbers.[52] The word "defect" seems to highlight an apparent disdain, bordering on moral apprehension to negative numbers, which is particularly fascinating as he effectively used negative numbers in his works.

Additionally, his namesake contribution to mathematics, the Cartesian plane also emphasizes Descartes' lack of a geometric understanding of negative numbers that Girard held. The term "Cartesian plane" as we use it today is somewhat problematic. Although Descartes designed a grid based coordinate system, his only included the positive quadrant, that is the quadrant where both the x and y coordinates are positive.[53] A more appropriate name for the plane used today would be the Reyneaudian plane, as it was Charles-René Reynaud who first published the four quadrant plane which used negative coordinates in *Analyse Demonstre*é in 1708, over 70 years after Descartes' *La Géométrie*.[54] By juxtaposing these two planes, we come to understand early seventeenth-century Catholic ideas on negative numbers: useful fictions related to number, but inexpressible in the spatial, geometric context that they are used in today.

Now knowledgeable on the world he lived in and the discursive rules he had to play by, we return our attention to the work of Albert Girard—professional lute player and amateur mathematician. His *New Discover[ies] in Algebra* have been surpassed by work in abstract algebra with the creation of proofs for his unbacked claims, but where the work has lost its cutting-edge mathematical use, it's gained a new historical one.

A Summary of *A New Discovery in Algebra*

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[51] Ibid.
[52] Ibid.
[53] Ibid., 49.
[54] Ibid., 81–82.

Published in Amsterdam in 1629, *Invention Nouvelle en l'Algèbre* is a treatise that deals with a variety of math problems. The pamphlet can be grouped into four sections: the dedication, an introduction on arithmetic, a section on algebra, and a section on geometry.[55] As the dedication will be discussed promptly in the beginning of my analysis of the pamphlet, it feels appropriate to begin summarizing after Girard's dedication. In his eight-page arithmetic section, Girard explains the common operations of addition subtraction, multiplication, and division with both whole numbers and fractions. The algebra section contains a much wider range of content than the arithmetic section and is correspondingly longer, spanning 41 pages. Within this segment of the work, Girard covers exponents, variables, solutions to cubic equations and more. The final section on geometry is 15 pages long, during which Girard primarily discusses spherical geometry. Among modern mathematicians, the work is renowned for being the first known publication that concretely states the fundamental theory of algebra.[56] Much like mathematics papers today, his pamphlet did not exist in a vacuum, but instead referenced, answered, and expanded on questions and ideas posed by other mathematicians, namely Diophantus, Simon Stevin, Gerolamo Cardano, and François Viète.

For Henry, With Love

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[55] The full title of the pamphlet is *A New Discovery in Algebra: Both for the Solutions of Equations, and for Recognizing the Number of Solutions that they Admit of, with Several Things Necessary for the Perfection of this Divine Science*.

[56] The fundamental theorem of algebra states that every non-zero, single-variable, degree n polynomial with complex coefficients has, counted with multiplicity, exactly n complex roots. In more accessible terms, this means that an equation has as many solutions as the degree of the highest exponent. Hence, in the equation $9x^8 - 4x^7 = 0$ would have eight solutions. These solutions can be repeated. The equation $x^3 + x^2 - 5x + 3 = 0$ has three solutions, $1, 1$, and -3. This is because $x^3 + x^2 - 5x + 3 = 0$ can also be written as $(x + 3)(x - 1)(x - 1) = 0$. As there are two $(x - 1)$ terms, 1 is a solution twice. The fundamental theorem of algebra had been stated before by Peter Roth in his 1608 book *Arithmetica Philosophia*, but Roth stated that a polynomial of degree n "may" have n roots. Girard stated that all polynomials of degree n have n roots, but also qualified his statement, which will be fully discussed later on. Readers of this paper do not need to be able to solve these equations to understand this paper. In many cases, computers are required to find or approximate these solutions. The only thing this paper requires is the belief that solutions to these equations can be positive, negative, or complex numbers.

While it's tempting to study Girard's views of negative numbers by analyzing only the mathematical sections of his work, we begin with Girard's dedication to understand what inspired him to write this pamphlet. Girard dedicates his *New Discovery* to Henry of Bergaigne, a calvary captain of the Netherlands, who apparently had excelled in mathematics.[57] He then summarizes his pamphlet, stating that it contains arithmetic, algebra, and geometry, three of the *purae* mathematical sciences of the day.[58] Particularly of note is his claim that some of the algebraic and geometric content is unknown "not only to the moderns, but also the ancients," alluding to some of his *New Discoveries*.[59]

This dedication is typical for its time. Authors used a variety of paratexts, varying from dedications and prefaces to artwork and front pieces in order to both thank their patrons for their support and to help raise math's position in the hierarchy of sciences.[60] While a calvary captain may seem like an odd benefactor for a pamphlet on mathematics, it's likely that Girard's praise of Henry of Bergaigne's mathematical prowess was genuine. During the Early Modern Period in France, military colleges served as centers of mathematical knowledge and debate.[61] In the increasingly technological world developing during the Scientific Revolution, military men needed knowledge of projectiles, fortifications, and geography, which could be gained at these colleges. Many mathematicians, including Albert Girard and Simon Stevin, who were focused on pure mathematics made their livings as military engineers. Although many consider mathematics a discipline removed from the violence of the physical world, conflict was

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[57]Albert Girard, *Invention Nouvelle en l'Algèbre*, trans. Ezra J. Marker, (Amsterdam, HO: Chez Guilliame Iansson Blaeuw, 1629), 2–3, https://books.google.com/books?id=JMQoAAAAcAAJ&printsec=frontcover#v=onepage&q&f=false.
[58] Volker R. Remmert, "Antiquity, Nobility, and Utility: Picturing the Early Modern Mathematical Sciences," In *The Oxford Handbook of the History of Mathematics*, edited by Eleanor Robson and Jacqueline A. Stedall, (Oxford, UK: Oxford University Press, 2011), 537.
[59] Girard, *Invention Nouvelle en l'Algèbre*, 2–3.
[60] Remmert, "Antiquity, Nobility, and Utility: Picturing the Early Modern Mathematical Sciences," 538.
[61] Schubring, *Conflicts between Generalization, Rigor, and Intuition*, 63.

everywhere in a post–Reformation Europe, and for a calvary captain like Henry of Bergaigne, mathematical knowledge was essential for getting ahead.

As we scan the dedication for any inclusion of negative numbers, we see that Girard writes nothing about them. This lack of reference to negatives coupled with his specific views and ideas of other authors he deals with later on are conspicuous. He claims that his work is significant as it contains "some novelties in Algebra and Geometry, unknown not only to the moderns, but also to the ancients, and there is *nothing else* that strikes me as particular [emphasis mine.]"[62] In a world with no definitive answers on negative numbers, Girard's lack of reference to their importance would seem to imply that he's endorsing an orthodox view of their nonexistence. In fact, that is the exact opposite position Girard ultimately maintains. Further complicating matters, the four authors Girard explicitly refers to in the work, Diophantus, Stevin, Cardano, and Viète, all have different views of negative numbers. Between greater discursive trends and intentionally chosen references, Girard's omission of his stance on negative numbers appears to purposefully mislead the reader and downplay the role of negative numbers in his work.

But why would Girard intentionally hide negative numbers? As a Calvinist in Holland, he would not be bound to French Catholic discourse about numbers in discussions on their existence in nature or biblical significance. For him, the discourses that mattered most were the remnants of Ancient Greek authors and the contemporary mathematicians who occasionally dipped their toes into the pool of negative numbers. While he might have been personally freed from some of the constraints of negative denialism commonly found within the Catholic tradition, he still has reason to omit them from his dedication. As discussed above, Girard intends to write to an overwhelmingly Catholic audience. Accounting for the religion of his readers, Girard needed to shape his work in a way that would be both

[62] Girard, *Invention Nouvelle en l'Algebre*, 2–3.

palatable and doctrinally appropriate to the Church's policy of speculation. Girard's specific positionality as a Protestant writing to a French audience influenced both what kind of knowledge he could openly embrace, and the methods he could use to bring that knowledge to his audience.

For the Arithmetically Inclined

The math of *A New Discovery in Algebra* begins with the very basics. The first page of the arithmetic section discusses the names of powers of ten and is then followed by an explanation of the four basic mathematical operations. Everything is incredibly rudimentary, not even using the algebraic symbols for the operations later introduced by Girard. He finishes this brief section with an introduction of fractions, as well as his "rule of threes."[63] While Girard assumes his reader knows the names of numbers, their representing symbols, and can conduct basic addition and subtraction of single digit numbers, he builds everything else up from language. The section on arithmetic appears to be in line with its description in the dedication, providing no new revolutionary mathematical knowledge. He does not use or define negative numbers in this section, and all example problems he poses are written to prevent them from occuring. Within this section, it seems that Girard is continuing to evade the topic of negatives, possibly believing them to require a stronger base of mathematical knowledge than introductory arithmetic.

O Algebr-other, Where Art Thou?

The section on algebra finally brings negative numbers into the eyes of Girard's reader. He begins by defining certain mathematical notations for the reader, explaining the symbols he uses for radicals, exponents, and what we now consider variables. Math notation was not standardized at the time, which compelled authors to explain the meanings of symbols they used in their works. Girard then assigns

[63] Ibid., 5–12.

notation for addition and subtraction, defining addition to be "equivalent to saying 'and' or even 'more'" while minus is used in the "way that one says 3 francs minus 5 sous."[64] Although it could be that Girard used a linguistic analogy for one definition and simile for the other just to keep the language of the pamphlet varied, these definitions press on an important epistemological issue for his readers. "And" and "more" are both generalizable to real world situations, (you can theoretically always get more of tangible objects that exist); the antonyms of "not" and "less" run into some issues. How can someone physically have "less" than nothing? "Not" having something implies its existence to begin with. To help his readers navigate around these issues, he implicitly defines subtraction less abstractly than addition, allowing it only when the quantity being subtracted from is greater than the quantity being subtracted. From a definitional standpoint, it seems that Girard is leading his reader to believe that negative numbers don't exist.

Following his explanation of addition and subtraction, Girard introduces a new operation "=," which "signifies the difference between quantities in whichever way there is a difference."[65] In the following paragraph, Girard provides examples of how these symbols work. If A and B are two numbers, their ". . . sum is $\mathrm{A} + \mathrm{B}$, the difference is $\mathrm{A} = \mathrm{B}$ (or rather, if A is greater, we will say their difference is $\mathrm{A} - \mathrm{B}$) . . ." It seems that Girard is echoing a limited conception of subtraction listed above, as "-" seems only to be accepted when A is greater than B, and all potential operations only can output positive numbers, as negative numbers are yet to be introduced. Girard is seemingly hitting his reader over and over on the head with these implicit claims that negative numbers don't exist, further leading his reader to believe that he will not engage with them.

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[64] Girard, *A New Discovery in Algebra*, 108.

[65] Ibid. In modern notation, this is the same as $|A - B|$. For examples of this operation, $6 = 9$ would be equal to 3, while $10 = 4$ would be equal to 6.

Yet his denialism soon comes to an end. The first negative number Girard includes is relatively inconspicuous. In the following section of “the 4 conjugations of the + and – signs,” Girard provides a peculiarly formatted rule of addition:

$$\text{“For signs }\begin{bmatrix}\text{like}\\ \text{unlike}\end{bmatrix}\text{ take the }\begin{bmatrix}\text{sum}\\ \text{difference}\end{bmatrix}\text{ with the sign }\begin{bmatrix}\text{that is common}\\ \text{of the greater number}\end{bmatrix}.\text{”}$$ [66]

With a little bit of trial and error, we can decipher Girard’s rules: the sentence can be interpreted two ways, reading the unbracketed quantities when they are present, and selecting to read only the top or bottom entries when given a choice in brackets.[67] The discussion of signs sets us up for negative numbers to appear, and Girard delivers. He provides the reader with this string of additions:

$$\begin{array}{cccccccc} 3 & +11 & +28 & -13 & -5 & -6 & +3 & +5 \\ -5 & -4 & -40 & +19 & +17 & -7 & +8 & -5 \\ \hline -2 & +7 & -12 & +6 & +12 & -13 & +11 & \end{array}$$

While the -5 could be read as subtracting, the -2 is undeniably and explicitly a negative number. Now that Girard has introduced us to a new type of quantity, we ought to see an explanation for his understanding of the quantity: “Note that the sign precedes the number, and that for brevity we do not put any mark in front of the first number when it must have +.” Unfortunately for the historian looking for quick and easy definition on a silver platter, none is to be found here. Even though Girard has granted us the negative number that we have searched so hard for, he has nothing to say on its existence, or even what a “signed” number is.

Here we must question Girard’s text about what knowledge he provides to his reader, and what he omits. An observation of his rule on addition is a fruitful place to start. Although formatted in an unorthodox way for modern readers, he explains how to conduct addition with both positive and negative

[66] Ibid.

[67] Therefore, his two rules are “For signs like take the sum with the sign that is common” and “For signs unlike take the difference with the sign of the greater number.”

numbers. His rule implicitly includes numbers that are not integers, although he provides no examples of them in his addition. While he does provide the reader with these seemingly basic rules, he also presumes that they have a surprising amount of precursory knowledge. For starters, he gives his reader no definitions of what negative numbers are, as he has only defined "-" as an operation, not as a modifier for a number. Additionally, he assumes that his reader would know what a signed number is, as well as how they differ from positive and negative numbers generally.[68] We can also see the lack of 0 mentioned earlier in the addition $5 + (-5)$, leading us to question Girard's authorial aptitude.

These knowns and unknowns that Girard has given his reader demand analysis. At first glance, it seems his pamphlet is full of contradictions. Why would he go through the work of linguistically delineating something so basic as addition, while leaving out explanations of more complicated topics like negative numbers? What is the goal of his work? Is it that his *New Discovery in Algebra* was a new way of learning mathematics? That's doubtful. Although Girard may have had some knowledge in writing educational works from editing Simon Stevin's pamphlets, as well as the fact that the work includes practice problems, the lack of an explicit definition for negatives leads us to believe that this is not the case. Perhaps Girard assumed his readers would be comfortable with negatives on their own and felt no need to explain them. While this theory does account for the absence of a definition of negative numbers, it leaves us with a different set of questions. It's likely that Henry of Bergaigne, the calvary man to whom the book was dedicated to, had a knowledge of the math that expanded well past the basic operations of arithmetic. Explicitly defining those operations while leaving open questions of negative numbers could border on patronization of his patron. There's the chance that this omission could simply be an oversight by Girard, but to leave out crucial definitions in a work he knew would be printed would be an embarrassing mistake.

[68] A signed number is a number preceded by a plus or minus sign to indicate a positive or negative quantity, respectively.

So, what explains these discrepancies? I propose that while Girard may have wanted to express an abstract theory of numbers, this would have raised religious issues with his French audience and the Catholic Church they followed. Valuing his algebraic contribution to mathematics more than some philosophical dispute on the existence of negative numbers, Girard chose to avoid a debate which would have derailed his pamphlet—and his lack of engagement makes sense. The controversiality and denial of negative numbers around Europe for over 300 years shows that there was no quick fix that would convince his readers of their existence. Even if there was, it would also need to be sanctioned by the Catholic Church. Girard wouldn't have wanted to waste pages in a debate that, while somewhat instrumental to his argument, he simply did not have the power to make. Instead of attempting the impossible, he chooses to work around the issue instead of confronting it head on, opting to spread his work without asking questions that could potentially draw the Church's ire. Even if Girard was willing to run the risk of antagonizing the Church, he was a more of a mathematician than a natural philosopher; his training was about how to use numbers, not how to argue for their existence. As such, he focused on results that could be derived with these numbers, leaving an explicit debate about the existence of negative numbers for theologians.

Coming back to the pamphlet, we can see Girard continuing to skirt the discussion of what a negative number is, while simultaneously explaining how they can be used. In the section "Multiplication of the + and – signs," he clarifies to the reader that a negative multiplied by a negative is a positive.[69] As well as following his convention of explaining almost everything to his reader, this served as a necessary clarification to the rules of negative multiplication. Mentioned in the discussion of contemporary mathematicians, Gerolamo Cardano stipulated that a negative multiplied by a negative was a negative, proceeding from his belief of two discrete areas of positive and negative numbers where "nothing could

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[69] Girard, *A New Discovery in Algebra*, 109.

trespass its [negative number's] forces."[70] As Girard directly references Cardano in "*On postposited quantities in algebra*," we can assume that he was somewhat familiar with Cardano's work and ideas.[71] Interestingly, here he also provides a linguistic definition of "-" as a modifier, in contrast to the simile used in the definition of the operation. Stating that "it is proper to make two consecutive signs, but seldom something like $+-3$, which is worth -3; for the antecedent + does not change anything, but the antecedent – does, by contradicting what follows."[72] By considering "-" a contradiction, Girard quietly grants negative numbers the same level of significance that was awarded to broadly abstractable operations such as addition, sneakily raising negative number's status and applications for his readers.

The definition of "-" as "contradicting" leads our discussion to the topic of Girard's terminology on negative numbers. Here we find two currents, one of traditional language and Greco–Catholic belief that denied negative numbers, and another, of Girard's personal convictions and rejection of traditional discourse on negative numbers. Direct statements by Girard about negative numbers which use anti-existence terminology are absent from treatise. The closest he gets are negations of their nonexistence, such as when he writes "It should not be considered strange that I have set down some things above that are less than nothing."[73] Here we can see Girard coming close to the "absurd" terminology laid down by Diophantus before him, but his rejection of categorizing negative numbers as strange emphasizes how he rebuffs both traditional discourse as well as beliefs on the existence of negatives.

But even though Girard doesn't use anti-existence terminology towards them, his language on negative numbers isn't overwhelmingly positive.[74] In attempting a division by $-\sqrt{3}-\sqrt{2}+1$, (a negative number, approximately -2.146) Girard notes that this quantity is "less than nothing, yet we should not

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[70] Schubring, *Conflicts between Generalization, Rigor, and Intuition*, 45.
[71] Girard, *A New Discovery in Algebra*, 146.
[72] Ibid., 110.
[73] Ibid., 130.
[74] Ba-dum-tss.

despair of succeeding with it."[75] The assumption of negative numbers eliciting despair from the reader is fascinating, and calls to Descartes' ideas of "defective quantities," though Girard's work predates Descartes' by 8 years.[76] Heavily mediated by Girard's assumptions, we're able glimpse on how he believes his Catholic readers would have responded to negatives: hopelessly.

The passage raises questions—what specifically is eliciting despair? The negative number in and of itself? The unsettling realization that what the reader believed fake actually existed? Negative numbers being used as divisors? The use of negative numbers in general? It's doubtful that Girard would have waited this long to tell his reader not to despair about negative numbers in the abstract—he's given his readers plenty of negative numbers before them without consoling his reader's feelings. The existence hypothesis is weak as well. He's dodged the question of explaining negative numbers up until now, and he continues to do that until the very end of the algebra section. It seems that the despair is rooted not in mathematics, but in definitions and metaphysics.

Looking back at Girard's definitions for operations help elucidate the problem his reader faces. We can see that he does not provide a rigorous linguistic definition of division in his "Division of the + and – signs" section, instead stating that "division is nothing more than a mixture of multiplication and subtraction."[77] We're then led to his definition of subtraction in the arithmetic section. There, he provides the reader with a proverb that states "Having the whole and the part, the rest is known."[78] Returning to the division at hand, it's important to note Girard's choice to describe this operation using language instead of the symbols he's introduced, showing a need to build up greater arguments from language as

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[75] Girard, *A New Discovery in Algebra*, 116.
[76] Notably, Girard uses the word "defective" in *A New Discovery in Algebra*, but he applies it to incomplete equations instead of negative numbers. These incomplete equations will be discussed later, but the belief that certain mathematical objects as lacking or failing is quite interesting, especially when imagining their existence as divine objects or abstractions.
[77] Girard, *A New Discovery in Algebra*, 109.
[78] Girard, *Invention Nouvelle en l'Algèbre*, 7.

he did in his arithmetic section. To begin, he implicitly describes $-\sqrt{3}-\sqrt{2}+1$, as not "something," and subsequently claiming that it's "less than nothing."[79] Coupled with his definition of subtraction, we find ourselves at the crux of the matter. In line with division being a mix of multiplication and subtraction, Girard must "Hav[e] the whole and part." But of logical and metaphysical importance, how can someone have a part of something that is "not something" and "less than nothing?" With this in mind, Girard attempts to soothe the reader by telling them not to despair, helping them proceed in this equation and brush the illogicality under the rug. As discussed above, Girard isn't a theologian or philosopher. He's an engineer and a mathematician who's focused on getting solutions. While troubling to his readers, he briefly soothes their woes, ignores the issues stemming from negative numbers, and pushes onward towards the main purpose of the work: sharing the fundamental theorem of algebra.

Our discussion of anti-existence terminology towards negatives would not be complete without the inclusion of complex numbers.[80] While he hasn't told the reader yet, we know that Girard accepts negative numbers and uses anti-existence terminology for negatives only when denying their non-existence. With complex numbers, however, Girard chooses to co-opt this language and apply it to these somewhat mysterious quantities. Throughout the treatise we can find Girard referring to these numbers as "impossible,"[81] "unsayable,"[82] and paradoxically, "inexpressible."[83] He then proceeds to reject these numbers in his pamphlet's crown jewel: the fundamental theorem of algebra. In his qualified theorem, Girard declares that "Every algebraic equation *except* the incomplete ones admits of as many solutions as

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[79] Girard, *A New Discovery in Algebra*, 116.

[80] A complex number is a number of the form $a+bi$. For those not acquainted with this terminology, a and b are real numbers, and could be any number, including 1 to $-e^{\pi}$ to $\frac{-1}{3}$, and i denotes the imaginary unit, which is equal to $\sqrt{-1}$. An example of a complex number is $2-5i$.

[81] Girard, *A New Discovery in Algebra*, 125.

[82] Ibid., 109

[83] Ibid., 125.

the denomination of the highest quantity indicates [emphasis mine.]"[84] While the more mathematically inclined readers will notice that Girard's qualifier on incomplete equations doesn't directly throw out all complex solutions, he clarifies his thoughts to the reader, stating that incomplete equations lead to "solutions that cannot possibly exist . . . where the impossibility lies due to the defectiveness and poor construction of the equation."[85] It seems that he's made a disastrous caveat to his key contribution to algebra: in an odd emotional stance against incomplete equations, he's thrown the baby out with the bathwater.

At this point, it seems that Girard's innovations on number weren't all that revolutionary. Sure, he accepted negative numbers and avoided some hard questions with his Catholic audience, but at the same time, his *New Discovery* seems to fall flat. He's lost the generality so sought after by mathematicians due to his rejection of complex solutions, and with our ability to look back, Girard becomes one of the traditionalists standing in the way of mathematical progress.

Except, Girard doesn't reject complex solutions. While he has qualified his fundamental theorem of algebra, he also qualifies his qualification. He notes that "As for incomplete equations, they do not always have as many solutions. None the less, we should not fail to explicate the solutions that cannot possibly exist. . . ."[86] He then proceeds to express these "inexpressible" solutions: "In the same way, if $[x^4 = 4x - 3]$ . . . the four solutions will be $1,1,1+\sqrt{-2}, 1-\sqrt{-2}$ [edited for mathematical clarity.]"[87] In

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[84] Ibid., 139. An explanation for the fundamental theorem of algebra can be found in footnote 56. In Girard's pamphlet, a complete equation of degree n refers to a polynomial where all terms $x^n, x^{n-1}, \ldots, x^1$ are multiplied by non-zero coefficients, while an incomplete equation of degree n refers to a polynomial where at least one of the terms $x^n, x^{n-1}, \ldots, x^1$ has a coefficient of zero. This is best explained through example. The polynomial $7x^5 + x^4 - 3x^3 + 11x^2 - 18x^1 + 1$ would be complete because the terms $x^5, x^4, \ldots, x^0$ are all present and have non-zero coefficients. The equation $5x^3 - 2x^1 + 5$ would be incomplete, as there is no x^2 term, hence x^2 has a coefficient of zero.

[85] Ibid., 140. The Girard's claim that excludes incomplete solutions is incorrect in many ways. There are complete equations with no real solutions like $x^2 + x + 1 = 0$ and incomplete equations with only real solutions, like $x^2 - 1 = 0$.

[86] Ibid.

[87] Ibid., 141.

a span of two pages, Girard has managed to both state, contradict, and reaffirm the most important theorem in algebra.

What's going on? Girard, who has sneakily accepted negative numbers both linguistically and existentially throughout his pamphlet seems to be full of doubts about numbers that are often grouped with negatives.[88] He has acknowledged complex numbers but hasn't given them the same support that he's granted negative numbers. We've already dismissed the idea that this is because Girard is a bad mathematician and author, who simply forgot to provide his readers with a clear and coherent explanation of complex numbers. In the recent preceding sections, Girard took care to meticulously (although sometimes inadequately) lay out definitions of all the mathematical tools his readers will need. That's not to say that he says nothing of these complex equations: he does consider numbers like $\sqrt{-3}$ to be "enveloped," and notes that these impossible solutions do have value in their use of proving general rules, their aid in finding all potential solutions, and their utility in solving similar equations.[89]

Girard's contradictions can be explained by the intentionality of his work. He knows his audience and exploits an existing discursive structure to maneuver around boundaries defining what can and cannot exist. Religion and philosophy are at the center of all of this, for Girard's fundamental theorem of algebra means nothing if it's not disseminated and remembered. This requires his theorem and its results to be acceptable to the Church, otherwise all his work would be for naught. His specific positionality as a Huguenot has made him acutely aware of what is needed to have his work accepted within the culture of early seventeenth-century French audience: if numbers are unable to be found in the real world, then they must be the useful but "false roots" as termed by Descartes. We know that Girard can find negative numbers in the real world though physical analogs, but complex numbers exist as a further abstraction of structure. He has willingly chosen to co-opt traditional discourse about negative numbers, without limiting

[88] Mumford, "What's so Baffling about Negative Numbers? — A Cross-Cultural Comparison," 139–140.
[89] Girard, *A New Discovery in Algebra*, 140–141

or negating it as he did with negatives and applies this terminology strictly to complex numbers. In borrowing anti-existence language, we can infer that Girard is attempting to play both sides, allowing his new discovery to be accepted by the Church and orthodox mathematicians, while also freeing himself from the direct terminology of speculation that would weaken his authority as an author and discourage other mathematicians from following the path he's laid.

Confusingly for his Protestant identity, Girard seems to choose to dive further into Catholic discourse by engaging with the book of nature. When solving the equation $x^2 = 6x - 10$ which has the complex solutions $3 + \sqrt{-1}$ and $3 - \sqrt{-1}$, Girard notes that these numbers are "not in nature."[90] As discussed earlier, belief in the book of nature is accepted in Catholic belief, and antithetical to Protestants. This is the only explicit reference to nature in the work, stating where solutions are <u>not</u> found. What does Girard have to gain by referencing the book of nature? The phrase "not in nature" seems to be a convergence of anti-existence terminology and the book of nature; two discourses that Girard was not bound to, one of which was actively against Calvinism's practice of *sola scriptura*. Perhaps he just wanted to use a new term to deny the existence of complex numbers.

Or perhaps, Girard is master of walking the tightrope of Protestant–Catholic discourse. At first glance, it seems that Girard is damning himself for both Protestants and Catholics. The denial of an answer in the divine book of nature would be questionable at best for his Catholic audience. For himself, a Calvinist, any belief in the book of nature would run counter to *sola scriptura*, which would put him in hot water. Girard wouldn't do that. Nobody would. Both the required knowledge of religion and his ambition of spreading mathematical knowledge would have made him intimately aware of how he can convey ideas to his audiences. In fact, "not in nature" is the *perfect* phrase for him to use, allowing him to be accepted by Catholics and staying true to his Calvinist religion. For his Catholic readers, "not in nature" allows them

[90] Ibid., 145.

to continue their tradition of denialist terminology towards complex and imaginary numbers. By claiming that they can't be found in (the book of) nature, Girard affirms their nonexistence. However, for Girard and other Protestants, "not in nature" does not detract from his views of these numbers. Though *sola scriptura,* "nature" is completely physical, and refers to the secular nature around him, instead of a theological nature that God created as a book to interpret. While he has a geometric real-world analog for negative numbers that will be fully discussed in the following section, he believes there is no such analog for complex numbers. They also find themselves "not in nature," but in an entirely different way than his Catholic readers would understand.

While a potential explanation, this theory leads to one more question: How could Girard make sure that his Catholic and Protestant readers interpreted this quote in their own doctrinally appropriate ways? This was by far the easiest part for him—he had to do nothing at all. As mentioned before, the Church would have no method of recourse against Girard besides suppressing the book, which covers a topic that is neither here-nor-there within the Catholic tradition; all that mattered was controlling discourse around topics that could raise theological issues. If there was a doctrinally appropriate way to interpret something, readers were primed select that interpretation as it conformed to their world views and allowed for all parties to win. While neither a Calvinist nor a Catholic, the case of Isaac Newton (1642–1726/27) is emblematic of this interaction between scriptural soundness and questionably heretical beliefs.[91] Much of Newton's work was focused on the movement and physics of astral bodies, and as an Anglican, he was bound to the practice of *sola scriptura*. One prominent issue he faced was the apparent contradiction between heliocentrism, the belief that the earth orbits the sun and Joshua's long day, a biblical event in which "the sun stood still, and the moon stayed, until the people had avenged themselves

[91] Andrew Janiak, "The Book of Nature, The Book of Scripture," *The New Atlantis*, September 26, 2020, https://www.thenewatlantis.com/publications/the-book-of-nature-the-book-of-scripture.

upon their enemies."[92] The claim that the sun stayed still seems to imply that the sun is moving around instead of the earth, which was at odds with the heliocentrism Newton observed. Unwilling to damn himself by denying scripture, and equally unwilling to deny the heliocentrism that showed the well ordering of the universe, Newton mediated the Bible; while the events in the Bible were correct, they were written from the point of view of a regular person, instead of a divine figure who could truly comprehend the actions of God.[93] This strategy allowed him to bring his potentially controversial theories in line with observed knowledge, mediating science and his religion. Here, Newton's willingness to accept a doctrinally appropriate solution for his math problems is emblematic of Girard's readers. By playing on different meanings and assuming his readers would follow their paths of least resistance, Girard's language on nature and complex numbers was both meaningful and doctrinally accepted by both his Protestant and Catholic audiences.

We return to the issue of negative numbers. There is one piece of evidence yet to be wholly discussed in this analysis. Mentioned throughout this paper, as the algebra section of the work comes to an end, Girard requires a transition to the geometry section of his work. At last, he reveals the physicalist conception of negative numbers that shapes his pamphlet, telling his reader: "Up till now we have not yet explained the purpose that the minus solutions serve, when there are some. The minus solution is explicated in Geometry by retrograding; the minus goes backwards where the + advances."[94] The final leg of our analysis focuses on this quote.

Why would Girard wait to share this insight until the end of the algebra section? While infuriating to the historian of mathematics who is only concerned with a surface level analysis of Girard's views, the placement of his explanation at the end of the work is particularly revealing through a more narrative

[92] Josh. 10:13 (NKJV).
[93] Janiak, "The Book of Nature, The Book of Scripture."
[94] Girard, *A New Discovery in Algebra*, 145.

lens. Alongside being a good transition section, preparing his reader for the geometry of the following portion of the work, it also has importance in Girard's hidden story of negatives. Through burying his unorthodox theoretical lead, Girard is able to show the utility of negative numbers to the potentially denialist reader of his pamphlet. Though it's a math treatise, we can read a sort of B-plot that tells a story of negative numbers. First, our narrator Girard introduces us to the basics of arithmetic. He takes these basic rules and shows us a signed world, where positives and negatives are connected and interact in a multitude of ways through a variety of complex rules. He then gives us examples of these rules in action, and shows the power of these quantities, solving problems that neither the ancients nor his contemporaries could. Making sure that his readers don't find him too controversial, he sets a seemingly hard boundary on complex numbers, but also shows the applications of these truly impossible quantities. Finally, Girard's protagonist, the negative, is accepted. He explains that the negative number could be found in physical, natural world all along: all the reader must do is pick a point on a line and choose a direction. While the goal of the work is to show his new algebraic theorem, Girard, the military engineer wants to show the applications of his work, and therefore the existence of the negative number in the physical world. Through this, Girard truly reveals himself in his pamphlet: the combination of the beauty of general mathematics with the real-world applications of his results affirm both Girard's identity as a mathematician and an engineer.

To emphasize the importance of saving his geometric-physicalist conception of negative numbers, we can act like mathematicians and assume the counter factual: What if Girard had started off his work with a claim on the existence of negative numbers, and then proceeded to show their utility? While the work would likely be received the same way by those who already accepted the concept of negatives, those who did not would have been much more reluctant to accept his conclusions. A flagrant, conspicuous embrace of negative numbers would have raised potential issues with the Catholic Church, preventing the spread of his work. Though Girard requires has required his reader to use negative numbers, until now he has done so instrumentally to share the fundamental theorem of algebra.

He finishes the algebra section poetically, providing the reader an inclination problem on a proto–Reyneaudian plane, and using algebra instrumentally to show both the power and the applications of negative numbers.[95] Reception is key to mathematical work: If *A New Discovery in Algebra* somehow popped into existence in the middle of a forest, and lay unseen until it decayed, there would be no effect on the real, living body of knowledge that is the story of mathematics. In order to share his work, Girard is willing to sacrifice clarity and sneakily appears to compromise on certain epistemological issues to make his audience ask: what really was his *New Discovery in Algebra*?

"So Much Concerning the Conjugations of Signs."[96]

With that, we find ourselves both at the end of the algebra section and the end this paper. Although we've seriously engaged with numbers, it would be a mistake to say that this paper is strictly about negative and complex numbers. Albert Girard has proven himself not just an innovative mathematician, but an author willing to use a variety of techniques to maneuver around existing structures to create space for his mathematical discoveries and ideas. Doing so, Girard's single pamphlet contributed to an infinitely greater debate that spanned countless textbooks, treatises, lectures, arguments, papers, and people that led us to the mathematical truth we hold today. While mathematicians are inclined to use the language of assumption within their proofs, the use of a historical lens challenges these "assume[s]" in order to find out *why* we can make these assumptions. To leave these assumptions unquestioned is to erase a vibrant, turbulent, and exciting history of ideas, further stoking

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[95] While Girard quickly solves this problem in a few steps using a quartic equation, a geometric approach could involve finding the side lengths of a plethora of triangles to calculate the solution.
[96] Girard, *A New Discovery in Algebra*, 110.

the fire of scientific anachronism, teleological triumphalism, and narratives of a truly "objective" mathematics.

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